Program : **B.Tech**

Subject Name: **Structural Analysis-II**

Subject Code: **CE-503**

Semester: **5th**

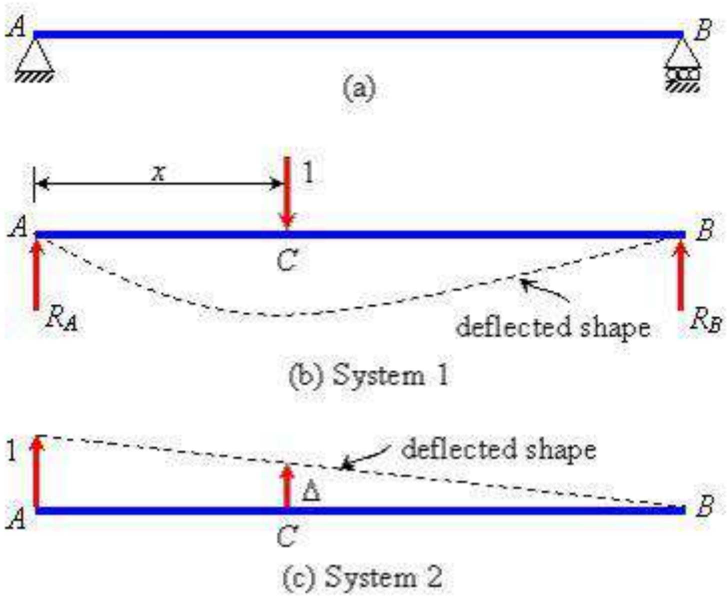
UNIT-V

INFLUENCE LINE DIAGRAMS

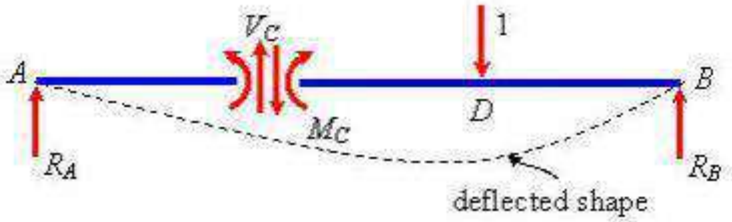
Influence lines for intermediate structures, Muller Breslau principle, Analysis of Beam-Columns.

Müller-Breslau Principle The Müller-Breslau principle uses Betti's law of virtual work to construct influence lines. To illustrate the method let us consider a structure AB. Let us apply a unit downward force at a distance x from A, at point C. Let us assume that it creates the vertical reactions and at supports A and B, respectively. Let us call this condition “System 1.”

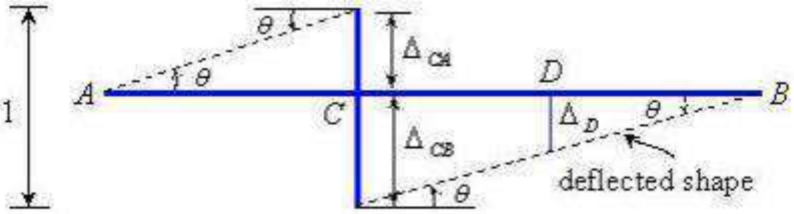
In “System 2” we have the same structure with a unit deflection applied in the direction of. Here is the deflection at point C.



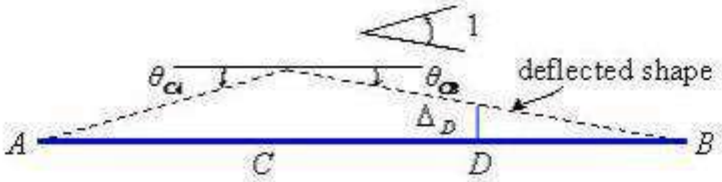
Given system AB, (b) System 1, structure under a unit load, (c) System 2, structure with a unit deflection corresponding to According to Betti's law, the virtual work done by the forces in System 1 going through the corresponding displacements in System 2 should be equal to the virtual work done by the forces in System 2 going through the corresponding displacements in System 1. For these two systems, we can write: The right side of this equation is zero, because in System 2 forces can exist only at the supports, corresponding to which the displacements in System 1 (at supports A and B) are zero. The negative sign before accounts for the fact that it acts against the unit load in System 1. Solving this equation we get: In other words, the reaction at support A due to a unit load at point C is equal to the displacement at point C when the structure is subjected to a unit displacement corresponding to the positive direction of support reaction at A.



(b) System 1



(c) System 2 for V_C



(d) System 2 for M_C

Similarly, we can place the unit load at any other point and obtain the support reaction due to that from System 2. Thus the deflection pattern in System 2 represents the influence line for. Following the same general procedure, we can obtain the influence line for any other response parameter as well.

$$(V_C)(\Delta_{CA} + \Delta_{CB}) + (1)(-\Delta_D) = 0$$
$$(V_C)(1) - \Delta_D = 0$$
$$V_C = \Delta_D$$

Let us consider the shear force at point C of a simply-supported beam AB. We apply a unit downward force at some point D as shown in System 1. In system 2 we apply a unit deflection corresponding to the shear force. Note that the displacement at point C is applied in a way such that there is no relative rotation between AC and CB. This will avoid any virtual work done by the bending moment at C ( ) going through the rotation in System 2.

$$(M_C)(\theta_{CA}+\theta_{CB})+(1)(-\Delta_D)=0$$
$$(M_C)(1)-\Delta_D=0$$
$$M_C=\Delta_D$$

Now, according to Betti's law:

The virtual work done by the forces in System 1 going through the corresponding displacements in System 2 should be equal to the virtual work done by the forces in System 2 going through the corresponding displacements in System 1. For these two systems, we can write: The right side of this equation is zero, because in System 2 forces can exist only at the supports, corresponding to which the displacements in System 1 (at supports A and B) are zero. The negative sign before accounts for the fact that it acts against the unit load in System 1.

In other words, the reaction at support A due to a unit load at point C is equal to the displacement at point C when the structure is subjected to a unit displacement corresponding to the positive direction of support reaction at A . Similarly, we can place the unit load at any other point and obtain the support reaction due to that from System 2. Thus the deflection pattern in System 2 represents the influence line for R_a .

Thus, the deflected shape in System 2 represents the influence line for shear force . Similarly, if we want to find the influence line for bending moment , we obtain System 2 by applying a unit rotation at point C (that is, a unit relative rotation between AC and CB). However, we do not want any relative displacement (between AC and CB) at point C in order to avoid any virtual work done by going through the displacements in System 2. Betti's law provides the virtual work equation:

Maximum shear at sections in a beam supporting two concentrated loads

Let us assume that instead of one single point load, there are two point loads P_1 and P_2 spaced at y moving from left to right on the beam as shown in Figure 1.1. We are interested to find maximum shear force in the beam at given section C. In the present case, we assume that P_2<P_1.

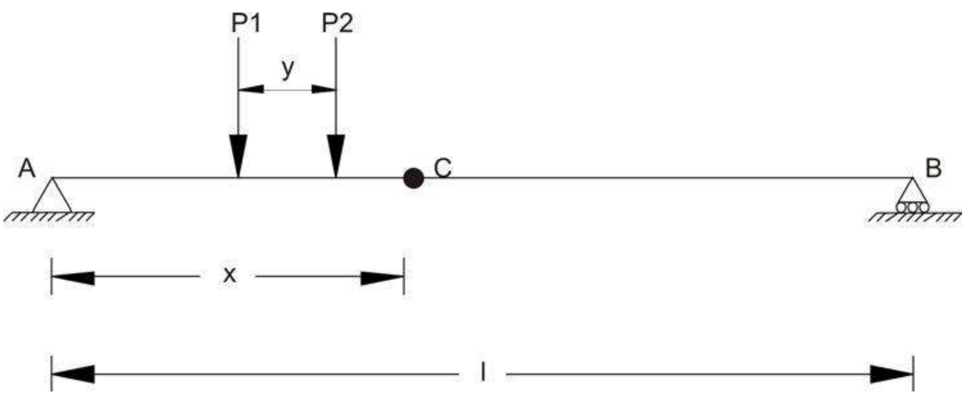


Figure 1.1: Beam loaded with two concentrated point loads

Now there are three possibilities due to load spacing. They are: x<y, x=y and x>y.

<u>**Case 1: x<y**</u>

This case indicates that when load P_2 will be between A and C then load P_1 will not be on the beam. In that case, maximum negative shear at section C can be given by

$V_C = -P_2 \frac{x}{l}$

and maximum positive shear at section C will be

<u>Case 2: x=y</u>

In this case, load P_1 will be on support A and P_2 will be on section C. Maximum negative shear can be given by

$V_C = -P_2 \frac{x}{l}$ and maximum positive shear at section C will be

<u>Case 3: x>y</u>

With reference to Figure 1.2, maximum negative shear force can be obtained when load P_2 will be on section C. The maximum negative shear force is expressed as:

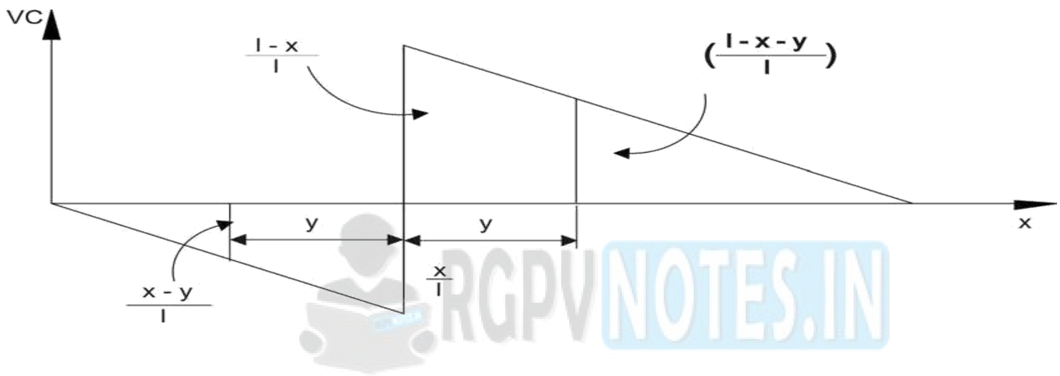


Figure 1.2: Influence line for shear at section C

$V_C^1 = -P_2 \frac{x}{l} - P_1 \frac{x-y}{l}$

And with reference to Figure 1.2, maximum positive shear force can be obtained when load P_1 will be on section C. The maximum positive shear force is expressed as:

$V_C^2 = -P_1 \frac{x}{l} + P_2 \frac{l-x-y}{l}$

From above discussed two values of shear force at section, select the maximum negative shear value.

Maximum moment at sections in a beam supporting two concentrated loads

Let us assume that instead of one single point load, there are two point loads P_1 and P_2 spaced at y moving left to right on the beam as shown in Figure 1.3. We are interested to find maximum moment in the beam at given section C.

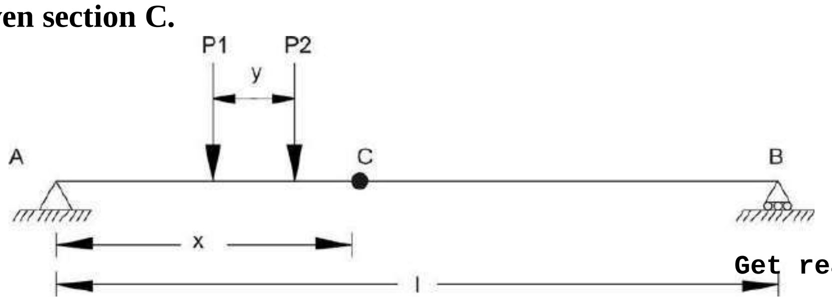


Figure 1.3: Beam loaded with two concentrated loads

With reference to Figure 1.4, moment can be obtained when load P_2 will be on section C. The moment for this case is expressed as:

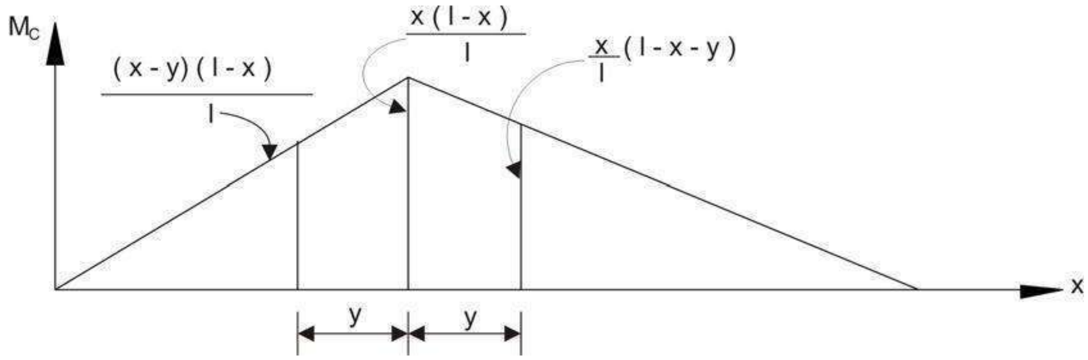


Figure 1.4: Influence line for moment at section C

$$M_C^{\,1} = P_1 (x-y)\frac{l-x}{l} + P_2\, x\frac{l-x}{l}$$

With reference to Figure 1.4, moment can be obtained when load P_1 will be on section C. The moment for this case is expressed as:

$$M_C^{\,2} = P_1\, x\frac{l-x}{l} + P_2\, x\frac{l-x-y}{l}$$

From above two cases, maximum value of moment should be considered for maximum moment at section C when two point loads are moving from left end to right end of the beam.

Maximum end shear in a beam supporting a series of moving concentrated loads

In real life situation, usually there are more than two point loads, which will be moving on bridges. Hence, in this case, our aim is to learn, how to find end shear in beam supporting a series of moving concentrated loads. Let us assume that as shown in Figure 1.5, four concentrated loads are moving from right end to left end on beam AB. The spacing of the concentrated load is given in Figure 1.5.

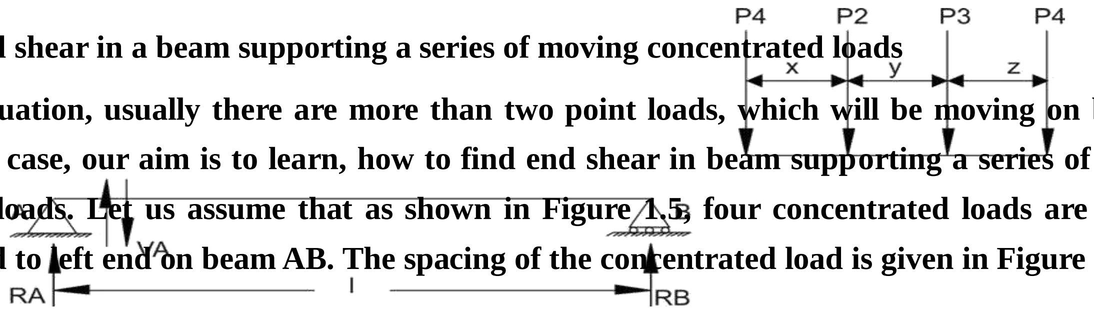


Figure 1.5: Beam loaded with a series of loads

As shown in figure, we are interested in end shear at A. We need to draw influence line for the support reaction A and a point away from the support at infinitesimal distance on the span for the shear V_A. The influence lines for these cases are shown in Figure 1.6 and 1.7.

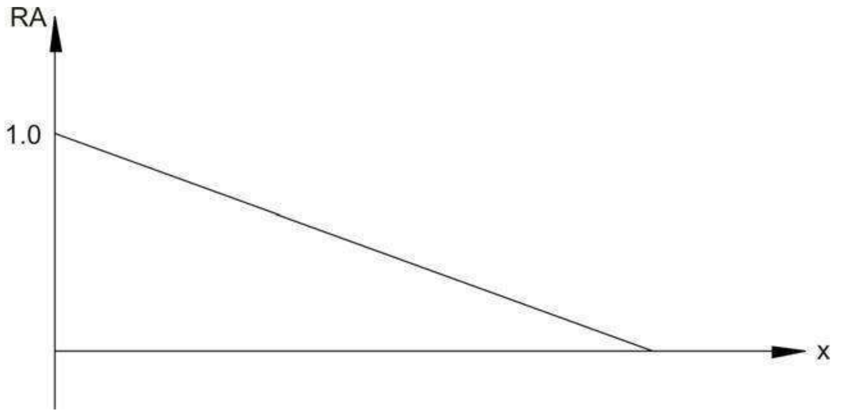


Figure 1.6: Influence line for reaction at support A

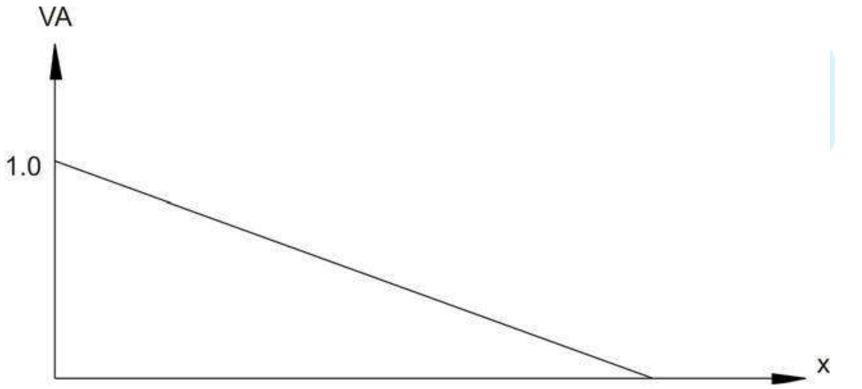


Figure 1.7: Influence line for shear near to support A.

When loads are moving from B to A then as they move closer to A, the shear value will increase. When load passes the support, there could be increase or decrease in shear value depending upon the next point load approaching support A. Using this simple logical approach, we will find out the change in shear value near support and monitor this change from positive value to negative value. Here for the present case let us assume that ΣP is summation of the loads remaining on the beam. When load P_1 crosses support A, then P_2 will approach A. In that case, change in shear will be expressed as

$$dV = \frac{\sum Px}{l} - P_1$$

When load P_2 crosses support A, then P_3 will approach A. In that case change in shear will be expressed as

$$dV = \frac{\sum Py}{l} - P_2$$

In case if dV is positive then shear at A has increased and if dV is negative, then shear at A has decreased. Therefore, first load, which crosses and induces negative changes in shear, should be placed on support A.

Numerical Example

Compute maximum end shear for the given beam loaded with moving loads .

When first load of 4 kN crosses support A and second load 8 kN is approaching support A, then change in shear can be given by

$$dV = \frac{\sum(8+8+4)2}{10} - 4 = 0$$

When second load of 8 kN crosses support A and third load 8 kN is approaching support A, then change in shear can be given by

$$dV = \frac{\sum(8+4)3}{10} - 8 = -3.8$$

Hence, as discussed earlier, the second load 8 kN has to be placed on support A to find out maximum end shear (refer Figure 1.9).

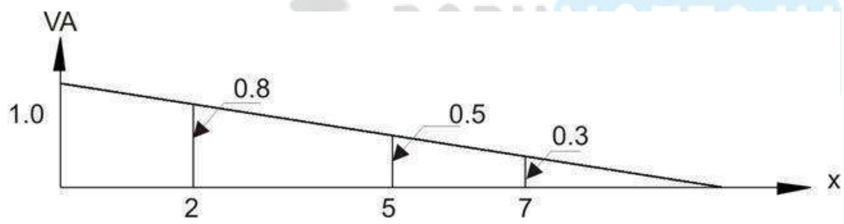


Figure 1.9: Influence line for shear at A.

$V_A = 4 \times 1 + 8 \times 0.8 + 8 \times 0.5 + 4 \times 0.3 = 15.6 kN$

Maximum shear at a section in a beam supporting a series of moving concentrated loads

In this section we will discuss about the case, when a series of concentrated loads are moving on beam and we are interested to find maximum shear at a section. Let us assume that series of loads are moving from right end to left end

Fig 1.10: Beam loaded with a series of loads

Monitor the sign of change in shear at the section from positive to negative and apply the concept discussed in earlier section. Following numerical example explains the same.

Numerical Example

The beam is loaded with concentrated loads, which are moving from right to left. Compute the maximum shear at the section C.

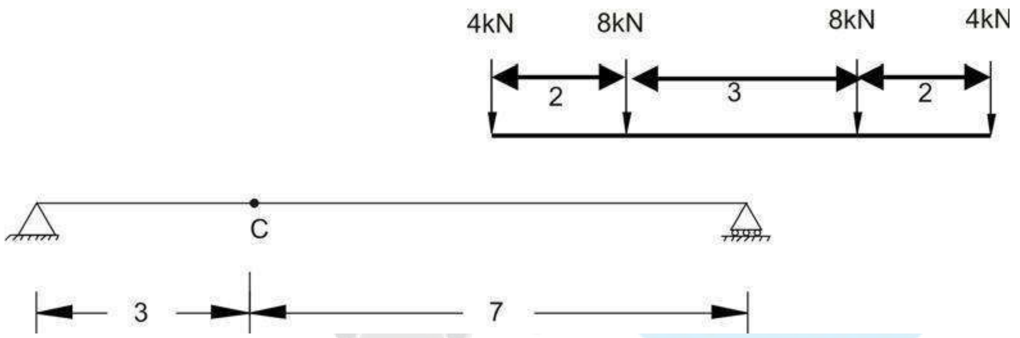


Figure 1.12: Beam loaded with a series of loads

The influence line at section C is shown in following Figure 1.13.

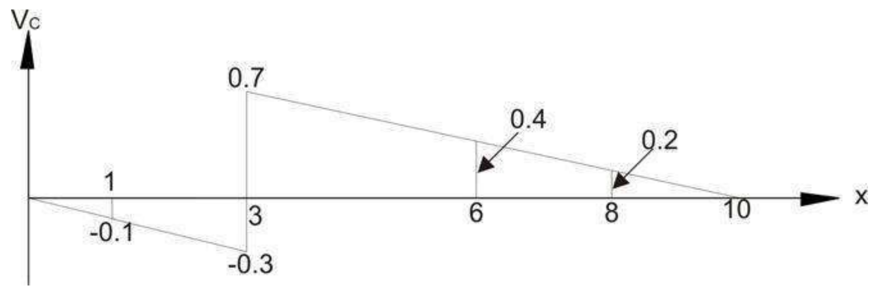


Figure 1.13: Influence line for shear at section C

When first load 4kN crosses section C and second load approaches section C then change in shear at a section can be given by

$$dV = \frac{20}{10}\times 2 - 4 = 0$$

When second load 8 kN crosses section C and third load approaches section C then change in shear at

section can be given by

$dV = \frac{12}{10}\times 3 - 8 = -4.4$

Hence place the second concentrated load at the section and computed shear at a section is given by

$V_C == 0.1\times 4 + 0.7\times 8 + 0.4\times 8 + 0.2\times 4 = 9.2kN$

Maximum Moment at a section in a beam supporting a series of moving concentrated loads

The approach that we discussed earlier can be applied in the present context also to determine the maximum positive moment for the beam supporting a series of moving concentrated loads. The change in moment for a load P_1 that moves from position x_1 to x_2 over a beam can be obtained by multiplying P_1 by the change in ordinate of the influence line i.e. $(y_2 - y_1)$. Let us assume the slope of the influence line (Figure 1.14) is S, then $(y_2 - y_1) = S(x_2 - x_1)$.

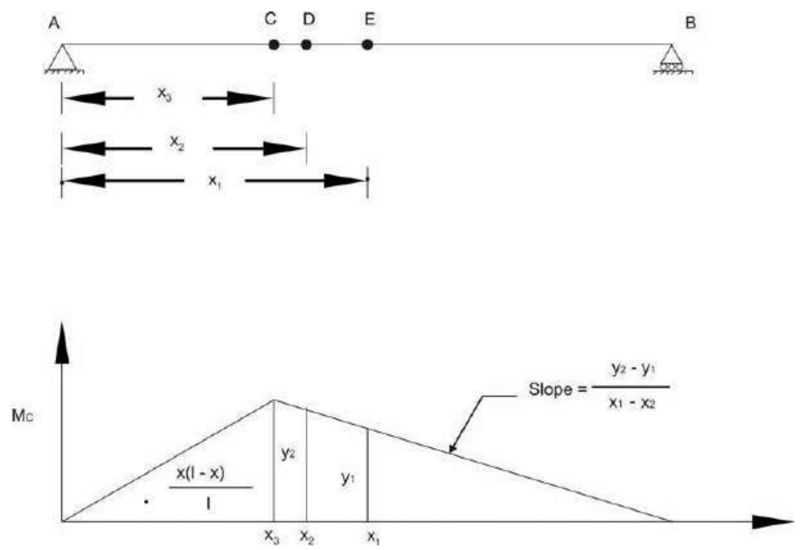


Figure 1.14: Beam and Influence line for moment at section C

Hence change in moment can be given by $dM = P_1 S(x_2 - x_1)$

Let us consider the numerical example for better understanding of the developed concept.